Multi-dof continued

recall: springs + masses: $M\ddot{\vec{X}} + K\vec{X} = \vec{0}$ ①

initial conditions: $\vec{X}(0) = \vec{X_0}$, $\dot{\vec{X}}(0) = \vec{V_0}$ ②

constants

How to solve ① and ②: Initial Value Problem

<u>Algorithm</u>

1. guess: $\vec{X} = \bar{X}e^{i\omega t}$ $\quad \vec{X}$ = constant

   Plug into ① from above

2. equation reads: $\underbrace{(-\omega^2 M + K)}_{A}\bar{X} = \vec{0}$

$$\det(A) = 0 \rightarrow \text{polynomial in } \omega^2$$

<u>Example:</u> 2x2 matrix → quadratic equation in ω^2

$W_1 = +\sqrt{\omega_1^2}$
$W_2 = +\sqrt{\omega_2^2}$ → positive real roots because M, K are positive matrices

3. For W_1, we solve $(-\omega_1^2 M + K)\bar{X} = 0$

   get $\bar{X}_1$

   For W_2, we solve $(-W_2^2 M + K)\bar{X} = 0$

   get $\bar{X}_2$

   etc. (for higher order matrices)

4. General Solution:

$\vec{X}(t) = C_1\bar{X}_1 e^{iW_1 t} + C_2\bar{X}_2 e^{iW_2 t} + \ldots$ for more $\bar{X}$'s

Real Solution

$$\boxed{= \bar{X}_1[C_{1c}\cos W_1 t + C_{1s}\sin W_1 t] + \bar{X}_2[C_{2c}\cos W_2 t + C_{2s}\sin W_2 t]}$$

⑤ Initial Value Problem

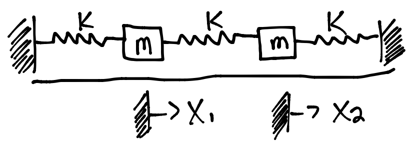
④ Implies $\bar{X}(0)$, $\dot{\vec{X}}(0)$

set $\vec{X}(0) = \vec{X_0}$, $\vec{V}(0) = \vec{V_0}$

4 equations for $C_{1s}, C_{1c}, C_{2s}, C_{2c}$

-Put in 8 numbers and got out 12

Example:



LMB:
$$m_1 : \quad m_1 \ddot{X}_1 + K_1 X_1 - K(X_2 - X_1) = 0$$
$$m_2 : \quad m_2 \ddot{X}_2 + K X_2 + K(X_2 - X_1) = 0$$

$$\underbrace{\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}}_{M} \ddot{\vec{X}} + \underbrace{\begin{bmatrix} 2K & -K \\ -K & 2K \end{bmatrix}}_{K} \vec{X} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

1.) guess $\vec{x} = \bar{X} e^{i\omega t} \rightarrow \det(-\omega^2 M + K) = 0$

$$\det \left| \begin{bmatrix} 2K - \omega^2 m & -K \\ -K & 2K - \omega^2 m \end{bmatrix} \right| = 0$$

$$(2K - \omega^2 m)^2 - K^2 = 0$$
$$4K^2 - 4K\omega^2 m + (\omega^2)^2 m^2 - K^2 = 0 \qquad \omega^2 \rightarrow \text{variable}$$

$$m^2 (w^2)^2 - 4Km\, w^2 + 3K^2 = 0$$

$$w^2 = \frac{4Km \pm \sqrt{16K^2m^2 - 12K^2m^2}}{2m^2} = \frac{K}{M}\left(2 \pm \sqrt{4-3}\right.$$

$$w_1 = \sqrt{\frac{3K}{m}}, \quad w_2 = \sqrt{\frac{K}{m}}$$

Solve equation 3 plug in one of the roots

$$w_1^2 = K/m \qquad \begin{bmatrix} K & -K \\ -K & K \end{bmatrix} \begin{bmatrix} \bar{X}_1 \\ \bar{X}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\bar{X}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$w_2^2 \rightarrow \bar{X}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

general solution : $$\boxed{\begin{aligned} \vec{X}(t) = &\begin{bmatrix} 1 \\ 1 \end{bmatrix}\left(C_{1c} \cos\sqrt{\tfrac{K}{m}}\,t + C_{1s} \sin\sqrt{\tfrac{K}{m}}\,t\right) \\ &+ \begin{bmatrix} 1 \\ -1 \end{bmatrix}\left(C_{2c} \cos\sqrt{\tfrac{3K}{m}}\,t + C_{2s} \sin\sqrt{\tfrac{3K}{m}}\,t\right) \end{aligned}}$$